Assessing Unconfoundedness

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Introduction

- Unconfoundedness assumption is that the probability of assignment is independent to the potential outcomes.

$$W \perp\!\!\!\perp Y(0), Y(1) | X$$

- The data cannot provide evidence of the unconfoundedness assumption.
- Consider ways to assess the plausibility of the assumption from the data at hand.

Introduction

- The specific methods are divided into three classes.
  - Design approach : Not use outcome.
  - Semi-design approach : Only use outcome in the control group.
  - Non-design approach : Use outcome.

Design approach

- Partition the full set of pre-treatment variables into two parts.
- The covariates X_i is divided to X_i^p,X_i^r so that $X_i = (X_i^p, X_i^r)$.
- X_i^p (Proxy variable for potential outcome) : Known a priori not to be affected by versus treatment control.
- Apply this method when X_i contains mutiple lagged measures of the outcome.

Design approach

- Instead of testing whether uncoundedness assumption holds directly, test whether the following conditional independence holds:

$$W_i \perp\!\!\!\perp X_i^p | X_i^r$$

- It implies that

$$\mathbb{E}\left[\mathbb{E}\left[g\left(X_i^p\right) \mid W_i = 1, X_i^r\right] - \mathbb{E}\left[g\left(X_i^p\right) \mid W_i = 0, X_i^r\right]\right] = 0$$

for any function g.

Design approach

- One might test jointly whether the effects of the treatment on $1_{X_i^{\mathrm{p}} \leq 0.2}, 1_{X_i^{\mathrm{p}} \leq 0.4}, 1_{X_i^{\mathrm{p}} \leq 0.6},$ and $1_{X_i^{\mathrm{p}} \leq 0.8}$ are all zero.
- Also not only on average, but conditional on $X_i^r = x^r$, for all x^r,

$$\mathbb{E}\left[g(X_i^{\mathrm{p}}) \mid W_i = 1, X_i^{\mathrm{r}} = x^{\mathrm{r}}\right] - \mathbb{E}\left[g(X_i^{\mathrm{p}}) \mid W_i = 0, X_i^{\mathrm{r}} = x^{\mathrm{r}}\right] = 0$$

- One can consider tests

$$\mathbb{E}\left[\mathbb{E}\left[g\left(X_i^{\mathrm{p}}\right) \mid W_i = 1, X_i^{\mathrm{r}}\right] - \mathbb{E}\left[g\left(X_i^{\mathrm{p}}\right) \mid W_i = 0, X_i^{\mathrm{r}}\right] \mid X_i^{\mathrm{r}} \in \mathbb{X}_j^{\mathrm{r}}\right] = 0$$

where $\mathbb{X}_j^{\mathrm{r}}$ is partition of the support $\mathbb{X}^{\mathrm{r}}$ of the set of X_i^r.

Semi-design approach

- This approach to assess the plausibility of the unconfoundedness assumption when there is a two-component control group.
- Let $G_i = \{c_1, c_2, t\}$ be an indicator denoting the treatment group that unit i is a member of.

$$W_i = \begin{cases} 0 & \text{if } G_i = c_1, c_2 \\ 1 & \text{if } G_i = t \end{cases}$$

- Instead of testing unconfoundedness, testing

$$G_i \perp\!\!\!\perp (Y_i(0), Y_i(1)) | X_i$$

- It has testable restrictions

$$G_i \perp\!\!\!\perp Y_i(0) \mid X_i, G_i \in \{c_1, c_2\}$$

which is equivalent to

$$G_i \perp\!\!\!\perp Y_i^{\text{obs}} \mid X_i, G_i \in \{c_1, c_2\}$$

Semi-design approach

- Given a partition $\{\mathbb{X}_j\}_{j=1}^{J}$ of the support $\mathbb{X}$ of X_i,

  $$\mathbb{E}\left[\mathbb{E}\left[g\left(Y_i^{\mathrm{obs}}\right) \mid G_i = c_1, X_i\right] - \mathbb{E}\left[g\left(Y_i^{\mathrm{obs}}\right) \mid G_i = c_2, X_i\right] \mid X_i \in \mathbb{X}_j\right] = 0,$$

  for all subsets $\mathbb{X}_j$, for $j = 1, \ldots, J$.

Non-design approach

- Consider subset unconfoundedness assumption

$$W_i \perp\!\!\!\perp Y_i(0), Y_i(1) | X_i^r$$

- Under the subset unconfoundedness and unconfoundedness, average causal effect which adjusts in the subset of covariates X_i^r and in the full set of covariates X_i should give similar result.

Non-design approach

- Let $\mathbb{X}^{\mathrm{r}}$ be the support of X_i^{r}.
- Let $\mathbb{X}_1^{\mathrm{r}}, \ldots, \mathbb{X}_J^{\mathrm{r}}$ be the partition of $\mathbb{X}^{\mathrm{r}}$.
- Under unconfoundedness

$$\begin{aligned}
&\mathbb{E}\left[g\left(Y_i^{\mathrm{obs}}\right) \mid W_i = w, X_i^{\mathrm{r}} \in \mathbb{X}_j^{\mathrm{r}}\right] \\
&= \mathbb{E}\left[\mathbb{E}\left[g\left(Y_i^{\mathrm{obs}}\right) \mid W_i = w, X_i\right] \mid W_i = 0, X_i^{\mathrm{r}} \in \mathbb{X}_j^{\mathrm{r}}\right]
\end{aligned}$$

- Under subset unconfoundedness

$$\begin{aligned}
&\mathbb{E}\left[g\left(Y_i^{\mathrm{obs}}\right) \mid W_i = w, X_i^{\mathrm{r}} \in \mathbb{X}_j^{\mathrm{r}}\right] \\
&= \mathbb{E}\left[\mathbb{E}\left[g\left(Y_i^{\mathrm{obs}}\right) \mid W_i = w, X_i^{\mathrm{r}}\right] \mid W_i = 1, X_i^{\mathrm{r}} \in \mathbb{X}_j^{\mathrm{r}}\right]
\end{aligned}$$

for all g, for all subsets $\mathbb{X}_j^{\mathrm{r}}$ and for $w = 0, 1$.